Technical Notes

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Heat Transfer Due to Axial Turbulent Flow Along a Circular Rod

Fue-Sang Lien* and Cha'o-Kuang Chen†
National Cheng Kung University
Tainan, Taiwan, China
and
J. W. Cleaver‡
University of Liverpool, Liverpool, England

Introduction

In the conventional heat-transfer analysis of a fin, it is standard practice to assume that the heat-transfer coefficient for convection at the fin surface is uniform over the entire fin. This approach is generally inadequate, since the heat-transfer coefficient varies along the fin surface. Recently, Huang and Chen[1,2] have studied the problem of a vertical circular pin with conjugated forced and mixed convection-conduction flow and concluded, based on their numerical analysis, that the overall heat-transfer rate increases with decreasing radius of the cylindrical pin.

Although extensive investigations have been made for the case of laminar Newtonian fluid, there are fewer papers about the turbulent case. To the best knowledge of the authors, the only such studies reported for the problem of a vertical plate fin with conjugated forced convection-conduction turbulent flow are the numerical analyses of Lien et al.[3]

The present analysis is a numerical study of heat transfer due to turbulent flow along a vertical circular pin. The transition position was assumed to occur at $R_{xtr}=5\times10^5$. In this study, we use the eddy viscosity formulation for forced convection flow developed by Cebeci and Smith.[4] Numerical results are presented for $Pr=0.7$, $Prt=0.9$, $u_\infty=100$ m/s, $\nu=1.5\times10^{-5}$ m^2/s, $L=0.1$ m, and $r_0=0.5$ cm over a conjugated convection-conduction parameter of $N_c=0.25$, 0.75, 2, and 3.

Analysis

Consider a vertical circular pin that is extended from a wall at temperature T_0 and situated in a turbulent flowfield with undisturbed oncoming freestream velocity u_∞ and temperature T_∞. The coordinate system is given in Fig. 1. The boundary-layer equations and their boundary conditions are expressed, respectively, as follows:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}(\epsilon_m+\nu)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}(\epsilon_h+\alpha)\right] \tag{3}$$

$$\begin{aligned} &u=v=0, && T=T_w(x) && \text{at } r=r_0\\ &u=u_\infty, && T=T_\infty && \text{as } r\to\infty\\ &u=u_\infty, && T=T_\infty && \text{at } x=0,\ r\geq r_0 \end{aligned} \tag{4}$$

where ϵ_m and ϵ_h are the eddy viscosity and the eddy conductivity, respectively. We introduce the pseudosimilarity variables (ξ,η), dimensionless streamfunction $f(\xi,\eta)$, and dimensionless temperature $\theta(\xi,\eta)$ as follows:

$$\xi=\frac{x}{L}\qquad \eta=\frac{r^2-r_0^2}{2r_0L}\left(\frac{Re_L}{\xi}\right)^{1/2} \tag{5}$$

$$f(\xi,\eta)=\psi(x,r)/[r_0(u_\infty L\xi\nu)^{1/2}] \tag{6}$$

$$\theta(\xi,\eta)=(T-T_\infty)/(T_0-T_\infty) \tag{7}$$

where L is the length of the cylinder pin, the Reynolds number $Re_L=u_\infty L/\nu$, and the streamfunction $\psi(x,r)$ satisfies the continuity equation (1) with

$$ru=\frac{\partial\psi}{\partial r},\qquad rv=-\frac{\partial\psi}{\partial x} \tag{8}$$

Substituting Eqs. (5-7) into Eqs. (2-4), gives

Momentum

$$(1+\lambda\eta\xi^{1/2})[(1+\epsilon_m^+)f'']'+\left[\lambda\xi^{1/2}(1+\epsilon_m^+)+\frac{f}{2}\right]f'' =\xi\left(f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right) \tag{9}$$

Energy

$$(1+\lambda\eta\xi^{1/2})[(Pr^{-1}+\epsilon_h^+)\theta']'+[\lambda\xi^{1/2}(Pr^{-1}+\epsilon_h^+)+(f/2)]\theta' =\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{10}$$

$$\begin{aligned} &f=f'=0, && \theta=\theta_w(\xi) && \text{at } \eta=0\\ &f'\to1, && \theta\to0 && \text{as } \eta\to\infty \end{aligned} \tag{11}$$

In the foregoing equations, the primes stand for partial derivatives with respect to η, Pr is the Prandtl number, and λ the transverse curvature parameter defined as: $\lambda=2L/(Re_L^{1/2}r_0)$.

The thin-pin energy equation, associated boundary conditions, and basic thermal coupling conditions are the same as those in Ref. 1. The dimensionless heat conduction equation

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*Graduate Student, Department of Mechanical Engineering.
†Professor, Department of Mechanical Engineering.
‡Senior Lecturer, Department of Mechanical Engineering.

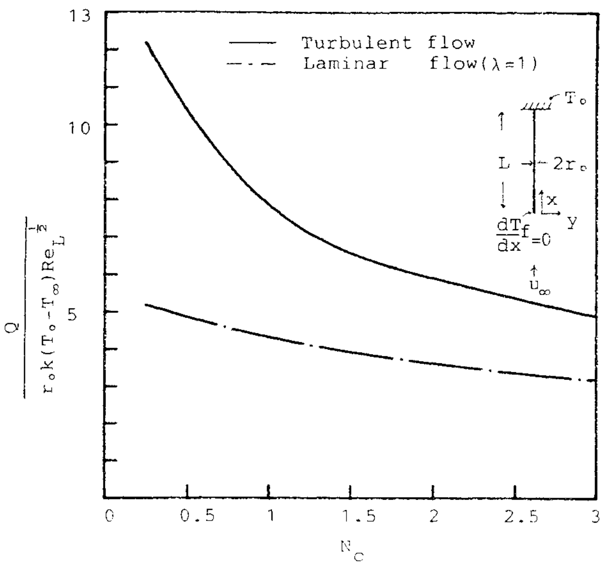

Fig. 1 Total heat-transfer rate for $Pr=0.7$, $Prt=0.9$, $\xi_{tr}=0.75$, and $\lambda=0.049$.

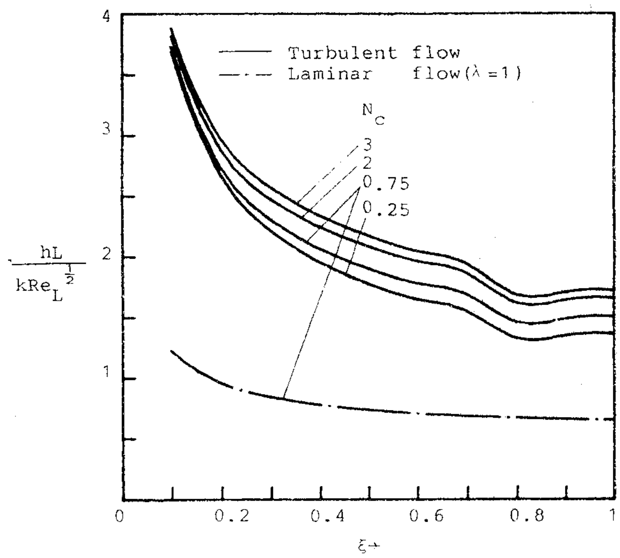

Fig. 2 Local heat transfer coefficient along the circular pin for $Pr=0.7$ and $Prt=0.9$, $\xi_{tr}=0.75$, $\lambda=0.049$, and various values of N_c.

and its boundary conditions are

$$\frac{d^2\theta}{d\xi^2}f = N_c \hat{h}\theta_f \tag{12}$$

and

$$\theta_f = 1 \text{ at } \xi=1, \quad d\theta_f/d\xi = 0 \text{ at } \xi=0 \tag{13}$$

where N_c is the conjugated convection-conduction parameter

$$N_c = 2kLRe_L^{1/2}/k_f r_0 \tag{14}$$

The quantity $\hat{h}$ is a dimensionless form of the load heat-transfer coefficient that can be obtained with Eq. (5) and the thermal coupling condition in Ref. 1,

$$\hat{h} = -\left(\frac{\partial\theta}{\partial\eta}\right)\bigg/(\theta_f\xi^{1/2}) \text{ at } \eta=0 \tag{15}$$

Eddy Diffusivity Formulas

Cebeci and Smith[4] reported an algebraic eddy viscosity formulation that is simple and effective for external wall boundary layer. According to this formulation, ϵ_m is defined by two separate formulas given by

$$\epsilon_m = \epsilon_{mi} = L^2\left(\frac{r}{r_0}\right)^{0.4}\left|\frac{\partial u}{\partial r}\right|\gamma_{tr}, \qquad \epsilon_{mi}\le\epsilon_{mo}$$

$$= \epsilon_{mo} = 0.0168\left|\int_{r_0}^{\infty}(u_\infty - u)\,dr\right|\gamma_{tr}, \quad \epsilon_{mi}\ge\epsilon_{mo} \tag{16}$$

where

$$L = 0.4\ln\left(\frac{r}{r_0}\right)\left\{1-\exp\left[-\frac{r_0}{A}\ln\left(\frac{r}{r_0}\right)\right]\right\} \tag{17}$$

and

$$A = 26\nu(\tau_w/\rho)^{-1/2} \tag{18}$$

In Eq. (16), γ_{tr} is an intermittency factor that accounts for the transitional region existing between a laminar and turbulent flow,

$$\gamma_{tr} = 1-\exp\left[-Gr_0(x_{tr})\left(\int_{x_{tr}}^{x}\frac{dx}{r_0}\right)\int_{x_{tr}}^{x}\frac{dx}{u_\infty}\right] \tag{19}$$

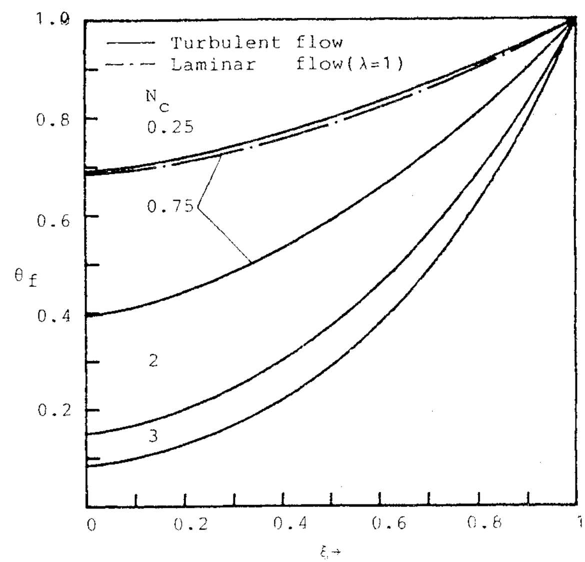

Fig. 3 Temperature distributions along the circular pin for $Pr=0.7$, $Prt=0.9$, $\xi_{tr}=0.75$, $\lambda=0.049$, and various values of N_c.

where x_{tr} is the location of the start of the transition and the empirical factor G is given by

$$G = u_\infty^3 R_{xtr}^{-1.34}/(1200\nu^2) \tag{20}$$

In terms of transformed variables, ϵ_m can be written as

$$\epsilon_m = \epsilon_{mi} = \nu\epsilon_{mi}^{+} = 0.16[r_0\ln(1+\lambda\eta\xi^{1/2})^{1/2}]^2 \times\left\{1-\exp\left[-\frac{u_\infty Re_x^{-1/4}f_w''^{1/2}r_0\ln(1+\lambda\eta\xi^{1/2})^{1/2}}{26\nu}\right]\right\}^2 \times(1+\lambda\eta\xi^{1/2})^{0.7}\frac{u_\infty^2 f'' Re_x^{-1/2}\gamma_{tr}}{\nu} \tag{21}$$

$$\epsilon_m = \epsilon_{mo} = \nu\epsilon_{mo}^{+} = 0.0168Re_x^{1/2}\nu r_0 \times\left[\int_0^\infty(1-f')\bigg/\sqrt{r_0^2+\frac{2r_0L\eta}{(Re_L/\xi)^{1/2}}}\,d\eta\right]\gamma_{tr} \tag{22}$$

We choose the turbulent Prandtl expression of Jischa and Rieke,[5] which is

$$Prt = \epsilon_m / \epsilon_h = a + b(Pr+1)/Pr \tag{23}$$

with $a = 0.825$ and $b = 0.0309$, a result fitting data for air ($Pr = 0.7$) quite well.

Results and Discussion

The numerical procedure of solving the boundary-layer equations and the heat conduction equation are identical to that in Ref. 1. The dimensionless overall rate of heat transfer Q from the pin can be expressed as

$$\frac{Q}{r_0 k(T_0 - T_\infty) Re_L^{1/2}} = \frac{2\pi}{N_c} \left. \frac{d\theta_f}{d\xi} \right|_{\xi=1} \tag{24}$$

or

$$\frac{Q}{r_0 k(T_0 - T_\infty) Re_L^{1/2}} = 2\pi \int_0^1 \left(-\frac{\partial \theta}{\partial \eta} \Big/ \xi^{1/2} \right)_{\eta=0} d\xi \tag{25}$$

The overall heat-transfer rate of the pin in the pure laminar flow and the mixed laminar and turbulent flow are shown in Fig. 1. It is observed from Fig. 1 that an increase in N_c yields a decrease in the corresponding overall heat transfer rate. Figure 1 also shows that the overall heat-transfer rate of the pin in the turbulent flow is higher than that of the pin in the laminar flow. This behavior is due to the action of turbulent eddies that increases the local heat transfer rate at the wall.

Figure 2 illustrates the local heat-transfer coefficient along the pin surface for various values of N_c. The local heat-transfer coefficients can be written in dimensionless form as

$$\frac{hL}{kRe_L^{1/2}} = -\frac{\theta'(\xi,0)}{\theta_f \xi^{1/2}} \tag{26}$$

As seen from Fig. 2, the distribution of the local heat-transfer coefficient $\hat{h}$ decreases monotonically before the transition zone along the streamwise direction, but the $\hat{h}$ becomes irregular in the transition zone due to the occurrence of the random spots of turbulence. This figure also shows that the larger values of N_c give rise to larger values of $\hat{h}$. It is observed from Fig. 2 that the turbulent-affected local heat-transfer coefficient is higher than $\hat{h}$ of the pin surface in pure laminar flow.

Figure 3 presents pin temperature distributions in turbulent forced convective flow. The figure illustrates that the larger values of N_c give rise to larger variations of pin temperature distributions. The phenomenon of this behavior is the same as the forced convective laminar flow over a circular pin.[1] Figure 3 also shows the the temperature distributions of the pin in the turbulent flow give a larger variation than those of the pin the laminar flow ($\epsilon_m, \epsilon_h = 0$). This behavior is attributed to enhanced surface heat-transfer rate associated with an increase in the random spots of turbulence along the streamwise direction.

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Pressure Fluctuations on Hypersonic Vehicles Due to Boundary-Layer Instabilities

A. Demetriades*
Montana State University, Bozeman, Montana

IT is generally agreed that a sequence of intense traveling waves appears in the hypersonic laminar boundary layer of cones in the region preceding the transition to turbulence.[1-7] Consisting of oscillations in the fluid velocity, density, and temperature, these waves are aligned with their fronts normal to the flow vector and are apparently due to the so-called second instability mode predicted by linear stability theory.[8] The connection of these "laminar waves," which represent the selective amplification of small ambient disturbances, to transition has been under experimental study for some time; their regularity and intensity, however, extends their possible importance beyond the issue of stability. For example, these waves may be dependable announcers of impending transition and may be ubiquitous and dominant features of the laminar portion of the flowfield, even for pitching or rolling (maneuvering) vehicles. Furthermore, because of the great strength of vorticity-generated pressure waves at hypersonic speeds, they may be important sound and vibration generators. These questions should be of interest to vehicle designers and are addressed in this Note.

Two recent experiments[9,10] performed in Tunnel B of the Arnold Engineering Development Center (AEDC) now provide interesting new results concerning the detection of the waves microphonically and also while the cone is pitching or rolling. Both tests were done in AEDC's Wind Tunnel B at stream Mach number $M_\infty = 8$ (edge $M_e = 7$) on slender, sharp-tipped cone models on which transition events were recorded over a wide variety of simulated "maneuvers"; these included both static and dynamic (continuous) changes in unit Reynolds number Re' and angle of attack α and also rolling motions at angle of attack. Attached flush on the surface of the models were a number of commercial pressure transducers acting as "surface microphones." Also installed within the structure of the models were a number of special "subsurface" pressure-sensitive transducers† separated from the airflow by a portion of the model structure skin that was 0.38 cm thick ATJ graphite in the first experiment[9] and carbon phenolic of thicknesses of 0.13-0.76 cm in the second.[10] This solid "shield" interposed over the subsurface microphones protected the latter from the hot fluid (and, in flight, from bombardment by ablation products); but it could also divert to the sensors acoustic signals from remote stations of the model through its structure and thus decrease their sensitivity to purely local events.

To test this possibility and to evaluate both the surface and the subsurface microphones as indicators of localized pressure fluctuations on the model, a variety of additional sensors were utilized for comparison. These consisted of the wind tunnel shadowgraph system, model surface pressure taps, surface thermocouples and heat-transfer (Gardon) gages, and externally supported probes such as pitot and pitot-acoustic (dynamic pitot) probes and hot-film anemometers. The surface and subsurface microphones and the film anemometer had a frequency response suitable for

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*Professor, Department of Mechanical Engineering.

†Designated as "boundary-layer acoustic monitors" or BLAM by their manufacturer, Kaman Sciences Corporation, Colorado Springs, CO.